Behavioural and Experimental Economics

Bilateral and multilateral bargaining
Lecture 8

December 3, 2014

The Bargaining Problem

Bargaining is a general name for a class of interactions between several (two or more) players aimed at allocation of some resources between them.

Examples include negotiations betwen spouses, neighbours, friends, workers and employers, firms sharing the market, countries etc.

Bilateral Bargaining

The simplest case is bilateral bargaining, for which there are many canonical games:

- Nash bargaining solution (1950) defines equilibrium by the product of excess utilities, $(u(x) - d(x)) \times (u(y) - d(y))$, subject to utility invariance, symmetry, Pareto and IIA axioms.

[1] Seller is uninformed but has all bargaining power, then $\max_p (1 - F(p))(p - c)$ results in $\frac{(1-F(p^*))}{f(p^*)} = (p^* - c)$ implying efficiency losses if $p^* > c$

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- Bargaining under unilateral[1] and bilateral (Myerson-Satterthwaite) incomplete information

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Cooperative games

A cooperative game is a pair $\langle N, v\rangle$, where N is the set of players, and $v : 2^{|N|-1} \to \mathbb{R}$ is the characteristic function that assigns payoff to every possible coalition.
Usually we consider transferable utility (TU) games, where payoffs can be transferred across coalition members; and superadditive games where $v(S \cup T) \geq v(S) + v(T)$ whenever $S, T \subseteq N$ satisfy $S \cap T = \emptyset$.
Example: left and right shoes game

Solution concepts

- The von Neumann-Morgenstern stable set is a set of imputations (allocations of the pie) $x, y\ldots$ such that x dominates y if some coalition $S \neq \emptyset$ satisfies $x_i > y_i, \forall\ i \in Sand \sum_{i \in S} x_i \leq v(S)$. In other words, players in S prefer the payoffs from x to those from y, and they can threaten to leave the grand coalition if y is used because the payoff they obtain on their own is at least as large as the allocation they receive under x. A stable set is a set of imputations that satisfies two properties:

  Internal stability: No payoff vector in the stable set is dominated by another vector in the set.

  External stability: All payoff vectors outside the set are dominated by at least one vector in the set.

- The core $C(v) = \left\{x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N); \quad \sum_{i \in S} x_i \geq v(S), \forall\ S \subseteq N\right\}$ — an imputation when no coalition has incentives to leave the grand coalition to get larger payoffs.
- The kernel $s_{ij}^v(x) = \max\left\{v(S) - \sum_{k \in S} x_k : S \subseteq N \setminus \{j\}, i \in S\right\}$, is the set of imputations where no player i has more bargaining power than player j in the sense that each player j is immune to player i's threats if $x_j = v(j)$, because he can obtain this payoff on his own.

Voting power: main notions

- N — set of agents (players), $|N| = n$, with generic player i
- $w_i > 0$ — number of votes i possesses
- q — quota (minimum number of votes for a bill to pass)

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- Coalition $S \subseteq 2^N$ is *winning* iff $\sum_{i \in S} w_i \geq q$ (denote $|S| = s$ and let W be the set of all winning coalitions)
- $v(S)$ — payoff to the coalition S. Let $v(S) = 1$ iff $S \in W$; $v(S) = 0$ iff $S \notin W$

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  (denote $|S| = s$ and let W be the set of all winning coalitions)
- $v(S)$ — payoff to the coalition S. Let $v(S) = 1$ iff $S \in W$; $v(S) = 0$ iff $S \notin W$
- Player $i \notin S$ is *pivotal* for the coalition S iff S is losing, while $S \cup \{i\}$ is not (thus, i is *decisive*)

Classical power indices

- Banzhaf (1965): $\beta_i = \frac{\sum_{S \subseteq N \setminus \{i\}} (v(S \cup \{j\}) - v(S))}{\sum_{j=1}^{N} \sum_{S \subseteq N \setminus \{j\}} (v(S \cup \{j\}) - v(S))}$
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- Shapley-Shubik (1954):
  $\phi_i = \sum_{S\subseteq N\setminus\{i\}} \frac{|S|!(N-|S|-1)!}{N!} (v(S \cup \{i\} - v(S))$.
  This is the share of permutations of all coalitions S in which player i is pivotal in the total number of permutations, i.e. the Shapley value for the cooperative voting game.

Preference-based power indices

(Aleskerov, 2006). Assume we know the preference profile of each player i about coalescing with any other player: $P_i = (p_{i1}, ..., p_{in})$.

Let p_{ij} be (ordinal or cardinal) *measure* of, or *explicit modifiers* of player i's preferences towards coalescing player j.

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Let $\chi_i = \sum_{S \subseteq N \setminus \{i\}} f_i(S) \, (v(S \cup \{i\}) - v(S))$ be the sum of intensities of connection of player i over all the winning coalitions in which she is pivotal.

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Similarly to the Banzhaf index, let

$$\alpha_i = \frac{\sum_{S \subseteq N \setminus \{i\}} f_i(S)(v(S \cup \{i\}) - v(S)))}{\sum^N \sum f(S)(v(S \cup \{i\}) - v(S))} = \frac{\chi_i}{\sum^N}$$

Power indices with preferences

Particular forms of the intensity of connections functions include

- $f_i^{\times}(S) = \prod_{j \in S \setminus \{i\}} p_{ij}$ — multiplicative intensity of i's preferences.
- $f_i^{\div}(S) = \prod_{j \in S \setminus \{i\}} p_{ji}$ — dual multiplicative intensity.
- ... and many others.

Experimental design (Montero, Sefton, Zhang, Soc Choice Welfare, 2008)

- Unstructured bargaining (Baron-Ferejoin 1989) game in groups of 3 or 4 players (12 or 16 participants per session).
- In each round of each game the players of a group decide on how to divide 120 points among them. Each player can post at most one offer at a time, and can vote for any offer on the board.
- The first offer to meet the quota is accepted, and the players receive the corresponding number of points unless they fail to come to an agreement within 300 seconds, in which case all receive 0 points.
- All players are randomly rematched from round to round.

Features of our experiment (Aleskerov, Belianin, Pogorelskiy, 2009)

- 2 games are played in each experimental session in randomized block order.
- With or without preferences (explicit modifiers).
- Games were played in Moscow, Perm and Tomsk in October 2008 - March 2010, using specially developed experimental software.
- Participants — over 500 students of various department, gender composition 50-50, average age 19.1 years.
- Gains of participants in 10-round games: average 7.62 EUR, minimum — 3.81 EUR, maximum — 13.68 EUR; gains in 20-round games: average 10.65 EUR, minimum 5.38 EUR, maximum 16.81 EUR per 1- to 1.5-hour session.

Screenshot of a typical game Standard (S)

Games S–1 (Standard) $[4; 3, 2, 2]$

Game S: quota is 4 votes

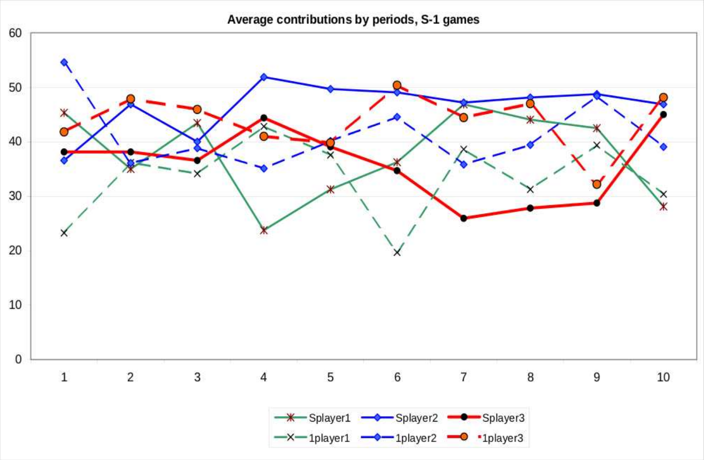
player#	1	2	3
votes	3	2	2

Winning coalitions: $W = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.
Banzhaf (and Shapley-Shubik) index: $\beta_1 = \beta_2 = \beta_3 = 1/3$, predicting that all players get around **40 pts** each. **Game 1** uses the following *explicit modifiers*:

	1	2	3
1	-	1	1
2	1	-	1.01
3	1	1	-

α indices based on the $f^{\div}$ intensity function:
$\alpha_1 = \alpha_2 = 0.3328, \alpha_3 = 0.3344$

The S-1 games


Average contributions by periods, S-1 games
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
Splayer1
Splayer2
Splayer3
1player1
1player2
1player3

The S-1 games

- Player 3 on average receives systematically more in the 1–treatment (43.85) than in S–treatment (35.84), which difference is significant. Hence **explicit modifiers work** for player 3: 'being loved is better than love'.
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- There are no treatment effects for players 1 and 2, but ...
- Player 2 receives systematically more than either of the other players in the S-treatment (46.5 vs. 35.84 or 37.66), the difference being significant.
  - Same effect as in MSZ, who attribute it to 'framing effect'
  - We specify: positioning of player 2 in the middle of the game table makes him receiving twice more offers from two immediate neighbours (1 and 3) than the other two players who have just one neighbour (player 2).

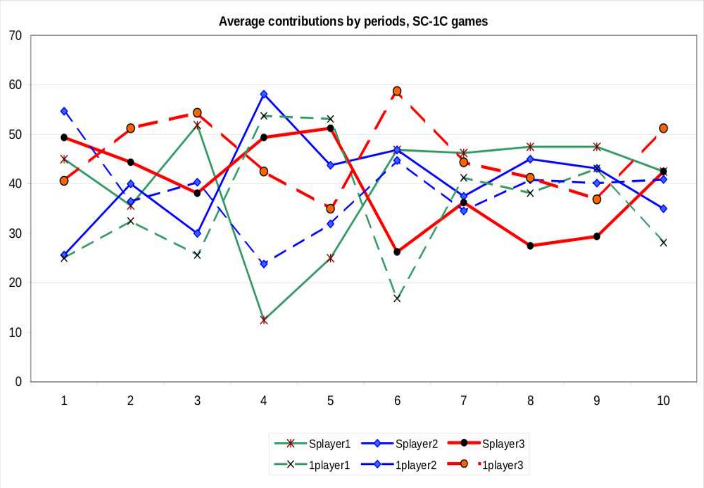
player#	1	2	3
votes	3	2	2
proposed shares	x	y	z

  - Effects in an **implicit modifier** to player 2's payoff

Way out: symmetric positioning

- In SC-1C games, each player is shown in the middle of the table in a systematic (clockwise) rotation.
- The difference between player 2 and the others in S-games is mitigated to (40.50 vs. 39.40 or 40.06), and becomes insignificant
  - We conjecture that the effect of *implicit modifier* is to completely disappear in a fully symmetric treatment.
- *Explicit modifiers'* effect persists for player 3, although to a somewhat smaller extent and over the last rounds.
- Average number of offers in games S (1) — 2.13 (resp., 2.42).
- Average time per round in games S (1) — 30 (resp., 37) seconds.

Results: SC–games



Summary of the S-1 games

All ($N = 320$)	mean	s.d.	min	max
player 1	35.36	29.04	0	80
player 2	44.53	24.42	0	100
player 3	40.1	27.56	0	111
Game S				
player 1	37.40	29.44	0	80
player 2	46.25	23.89	0	100
player 3	**36.34**	28.05	0	110
Game 1				
player 1	33.32	28.57	0	80
player 2	42.81	24.91	0	99
player 3	**43.85**	26.62	0	111

- No significant difference in payoffs for players 1 and 2.
- Significant difference for player 3 at 1-2% confidence level.
- Centered treatment suppresses implicit modifiers.

Games V–2 (Veto)

Game V: quota is 5 votes

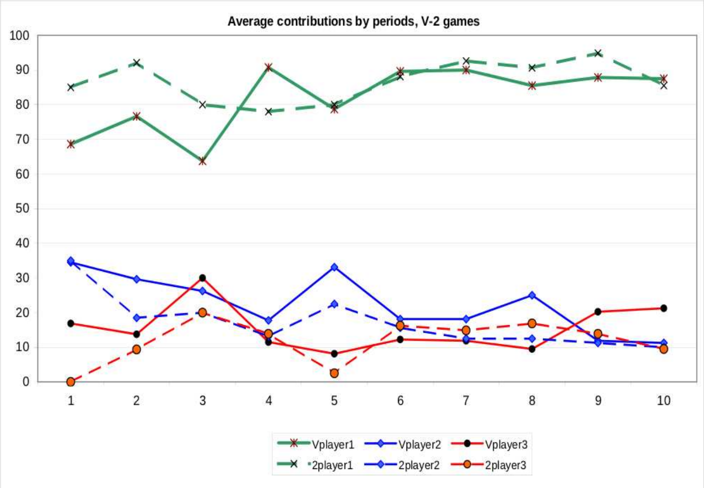
player#	1	2	3
votes	3	2	2

Winning coalitions $W = \{\{1,2\},\{1,3\},\{1,2,3\}\}$.
Banzhaf: $\beta_1 = 3/5, \beta_2 = \beta_3 = 1/5$, shares [72, 24, 24].
Shapley-Shubik: $\sigma_1 = 2/3, \sigma_2 = \sigma_3 = 1/6$, shares [80, 20, 20].
Game 2 uses the following *explicit modifiers*:

	1	2	3
1	-	1	1
2	0.99	-	1
3	0.99	1	-

α indices based on the $f^{\times}$ intensity function:
$\alpha_1 = 0.6016, \alpha_2 = \alpha_3 = 0.1992$

Results:V–2 games



Summary of the V-2 games

All ($N = 160$)	mean	s.d.	min	max
player 1	84.29	24.99	0	120
player 2	19.76	20.97	0	70
player 3	13.56	18.42	0	60
Game V				
player 1	81.90	24.76	0	119
player 2	22.56	23.40	0	70
player 3	15.53	19.99	0	60
Game 2				
player 1	86.68	25.14	0	120
player 2	16.96	17.94	0	60
player 3	11.60	16.59	0	60

Results: V–2 games

- Player 1 (the veto player) gets even more than the Banzhaf index predicts.
- No significant difference across treatments.
- Effects of greater negative modifiers might be larger.
- Average number of offers in games V (2) — 5.94 (resp., 5.63).
- Average decision time in games V (2) — 147 (resp., 141) seconds. Timing of decisions requires further attention.

Games E–3 (Enlarged)

Game E: Again, **5** votes are required to reach an agreement

player#	1	2	3	4
votes	3	2	2	1

Winning coalitions $W = \{\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\}$. Banzhaf (and Shapley-Shubik): $\beta_1 = 5/12, \beta_2 = \beta_3 = 3/12, \beta_4 = 1/12$, shares [50, 30, 30, 10].

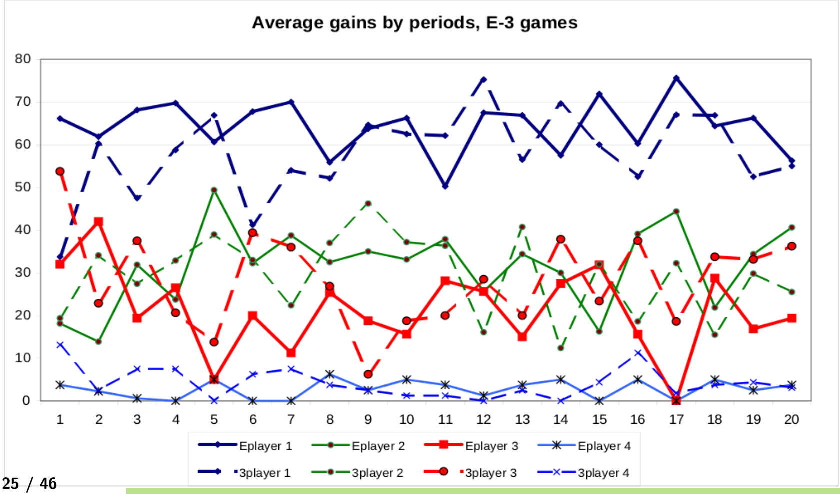
Game 3 employs the following modifiers:

	1	2	3	4
1	-	1	1	1
2	0.99	-	1	1
3	1	1	-	1
4	1	1	1	-

α indices based on the $f^{\times}$ intensity function:

$\alpha_1 = 0.5005, \alpha_2 = 0.1992, \alpha_3 = 0.2002, \alpha_4 = 0.1001$

The E–3 games


Average gains by periods, E-3 games
80
70
60
50
40
30
20
10
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Eplayer 1
Eplayer 2
Eplayer 3
Eplayer 4
3player 1
3player 2
3player 3
3player 4

Summary of the E-3 games

All ($N = 160$)	mean	s.d.	min	max
player 1	61.15	25.76	0	100
player 2	30.63	23.82	0	70
player 3	24.73	24.54	0	70
player 4	3.49	9.00	0	70
Game E				
player 1	**64.34**	22.36	0	95
player 2	31.65	23.17	0	70
player 3	**21.23**	23.72	0	70
player 4	2.76	7.40	0	40
Game 3				
player 1	**57.95**	28.47	0	100
player 2	29.59	24.48	0	70
player 3	**28.23**	24.90	0	65
player 4	4.21	10.33	0	70

Summary of the E-3 games

- In **E-game** player 1 gets systematically more than the Banzhaf index prediction at the expense of player 4, while gains of players 2 and 3 are in line with the index, and are *greater than in the V–2 treatment*.

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  - Thus, a small negative modifier towards player 1 indirectly benefits player 3, (gain per session increases by 25%).
- Frequency of coalitions $\{2, 3, 4\}$ is $\times 2$ higher in the 3–game than in the E–game.
  - Means that players 2, realizing they do not like player 1, tend to switch to a larger coalition, even though it is clearly more difficult and may involve lowering one's share of the pie (has to be divided among 3 players instead of 2).

Games F–4

Game F: 6 votes required to reach an agreement

player#	1	2	3	4
votes	3	3	2	2

Winning coalitions $W = \{\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},$ $\{2,3,4\},\{1,2,3,4\}\}$. Banzhaf index is $\beta_1 = \beta_2 = 1/3, \beta_3 = \beta_4 = 1/6$, 1 and 2 get 40, 3 and 4 get 20.

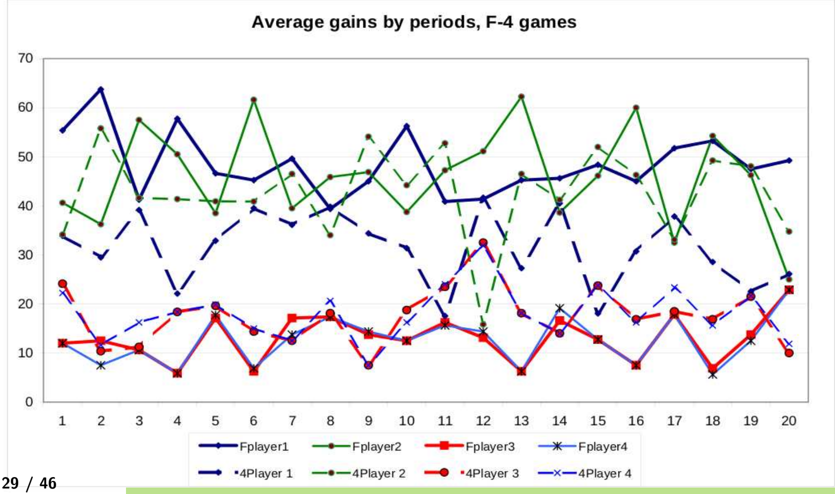
Game 4 employs the following modifiers:

	1	2	3	4
1	-	0.8	1	1.01
2	0.8	-	1	1.1
3	1	1	-	1
4	1	1	1	-

α indices based on $f^{\times}$ intensity function:

$\alpha_1 = 0.3107, \alpha_2 = 0.3583, \alpha_3 = \alpha_4 = 0.2002.$

The F–4 games


Average gains by periods, F-4 games
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Fplayer1
Fplayer2
Fplayer3
Fplayer4
4Player 1
4Player 2
4Player 3
4Player 4

Summary of the F–4 games

All ($N = 160$)	mean	s.d.	min	max
player 1	39.95	10.97	17.55	63.75
player 2	44.32	9.68	15.81	62.25
player 3	15.24	5.75	5.88	32.50
player 4	15.35	5.87	5.63	31.88
Game F				
player 1	**48.43**	6.34	39.38	63.75
player 2	45.97	10.02	25.00	62.25
player 3	**12.95**	4.68	5.88	22.88
player 4	**12.66**	4.95	5.63	22.88
Game 4				
player 1	**31.48**	7.46	17.55	41.66
player 2	42.67	9.30	15.81	55.80
player 3	**17.53**	5.90	7.50	32.50
player 4	**18.04**	5.58	7.50	31.88

Results: F-4 games

- 'Large' negative modifier of player 2 for player 1 significantly lowers her earnings (48.43 vs 31.48).
- On the contrast, player 2's payoff does not change much.

Results: F-4 games

- 'Large' negative modifier of player 2 for player 1 significantly lowers her earnings (48.43 vs 31.48).
- On the contrast, player 2's payoff does not change much.
- Complex interaction of modifiers: high 'dislike' modifiers of 0.8 tend to hurt player 1 more than player 2 because player 2 more strongly prefers larger coalitions.
- We also investigated another explanation – that the *psychological features* of the subjects' characters essentially influence their behaviour.

Coalitional outcomes across treatments

coalitions ＼ games	S–1 games			V–2 games	
S-1 coalitions	S	1	V-2 coalitions	V	2
1&2	54	33	**1&2**	41	40
2&3	29	33	**2&3**	27	26
2&3	56	59	1&2&3	12	10
1&2&3	21	35	1 alone	0	1
other	0	0	none	0	3
total	160	160	total	80	80

coalitions ＼ games	E–3 games			F–4 games	
E-3 coalitions	E	3	F-4 coalitions	F	4
1&2	73	74	**1&2**	82	64
2&3	57	51	**1&3&4**	38	31
2&3&4	13	26	**2&3&4**	33	56
1&2&3	5	1	1&2&3	1	1
1&2&4	1	3	1&2&4	1	0
1&3&4	1	1	1&3	0	1
1&2&3&4	9	3	1&4	0	0
none	1	0	1&2&3&4	4	6
total	160	160	total	160	160

Offers by roles, S-1 games

player	observations	mean	std.dev
all coalitions			
1	1155	35.46	28.86
2	1155	43.02	24.97
3	1155	41.51	27.12
$\{1,2\}$ coalitions			
1	196	61.74	7.48
2	196	58.25	7.48
3	196	0	0
$\{1,3\}$ coalitions			
1	166	59.88	10.47
2	166	0	0
3	166	60.12	10.47
$\{2,3\}$ coalitions			
1	274	0	0
2	274	59.26	4.51
3	274	60.73	4.51

Offers by roles, F-4 games

player	observations	mean	std.dev
all coalitions			
1	1280	42.59	27.66
2	1280	46.01	25.82
3	1280	15.49	15.00
4	1280	15.53	15.25
$\{1,2\}$ coalitions			
1	580	60.26	3.72
2	580	59.75	3.72
3	580	0	0
4	580	0	0
$\{1,3,4\}$ coalitions			
1	276	60.65	10.30
2	276	0	0
3	276	29.83	5.17
4	276	29.52	5.41
$\{2,3,4\}$ coalitions			
1	356	0	0
2	356	61.73	8.08
3	356	29.01	4.42
4	356	29.26	4.61

Composition of the winning coalition explains over 90% of shares' variations!

Typical bargaining process, S-games

$x_0 = 0, 0, 0$
$x_1^1 = 100, 10, 10$ (0 seconds, initial offer by player 1)
$x_3^2 = 65, 0, 55$ (5 seconds, player 3 rejects offer by 1)
$x_2^3 = 0, 60, 60$ (0 seconds, player 2 makes a better offer to player 3)
$x_3^4 = 0, 60, 60$ (13 seconds, player 3 accepts offer by 2)
This bargaining process is intuitively clear, but qualitatively different from the logic of most of the literature!

Interpretation

- An overwhelming majority of the outcomes result in *minimal* winning coalitions.
- Explicit modifiers are of secondary importance; by contrast, people use simple heuristic strategies that are not captured by either classical or generalized power indices (in their present formulation).
- The best predictors for the model are players' roles and the composition of the winning coalition.
  - S-1 60-60 for all three winning coalitions
  - V-2 85-25 for the coalitions $\{1, 2\}$ and $\{1, 3\}$, the rest being 'noise'
  - E-3 70-50 for the $\{1, 2\}$ and $\{1, 3\}$ coalitions, and 50-50-20 for the $\{2, 3, 4\}$ coalition.
  - F-4 60–60 for the $\{1, 2\}$ coalition, and the 60–30–30 for the $\{1, 3, 4\}$ and $\{2, 3, 4\}$ coalitions.
- How can we describe this evidence?

Key paradigms

- Describe the behavior of players in a bargaining game in a non-structured experiment (unlike most approaches, such as Baron and Ferejohn, 1989; Eraslan and McLennan, 2013 or Drouvelis, Montero, Sefton, 2009, we do not want to impose particular bargaining protocol)
- Theory of social situations: noncooperative game in search of a cooperative solution (Greenberg, 1989; Monderer e.a., 1996; Chwe, 1998; Xue, 1999; Herning e.a., 2007).
- Communication games: values for cooperative games constrained on a graph (Myerson, 1977; van den Nouweland, 1993; Jackson and Wolinsky, 1996; van den Brink, 2009; Gonzales-Aranguena e.a., 2008)
- The Nash programme: provide a noncooperative foundation for cooperative games solution (Serrano, 2007).

Strategic Approach to Nonstructured Bargaining I

- Noncooperative approach (Eraslan, 2003): assume the following axioms hold

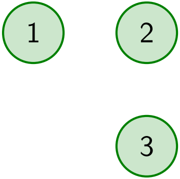
  Rationality: No player ever makes an offer that gives her less than she can get from any other existing offer

  Efficiency: $\sum_{i \in S} \psi_i(g) = v(S)$

  Improvement: No player can make an offer that worsens the stake of any of the players in any *minimal* winning coalition.

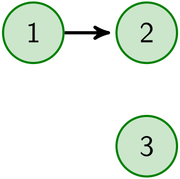
- **Proposition**: In an S–1 game with the Rationality, Efficiency and Improvement axioms, the only noncooperative Nash equilibrium is given by equal partitions within minimal coalitions.

Strategic approach II: dynamic incentives



Player 2 is priviledged after the first proposal is made to him, as he has more options (accept an offer, in addition to his offers), then offer of player 3 gives additional option to player 1 etc. All these developments change the Myerson value dynamically as the game goes on.

Strategic approach II: dynamic incentives



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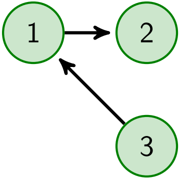

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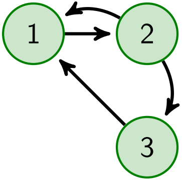

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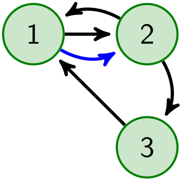

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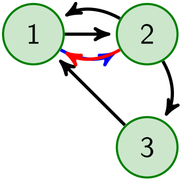


Player 2 is priviledged after the first proposal is made to him, as he has more options (accept an offer, in addition to his offers), then offer of player 3 gives additional option to player 1 etc. All these developments change the Myerson value dynamically as the game goes on.

The Myerson value

- $g \in G$ is a nondirected graph in the set of N players
- $S/g = \{\{i : i \text{ and } j \text{ are connected by } g\} | j \in S\}$ be the coalition constrained by g
- The Myerson value is a function $\psi : 2^N \times G \to \mathcal{R}^N$ satisfying

  Efficiency $\sum_{i \in S} \psi_i(g) = v(S)$

  Fairness $\psi_i(g \cup i) - \psi_i(g) = \psi_j(g \cup j) - \psi_j(g)$
- Myerson (1977) shows that the Myerson value is a unique function satisfying Efficiency and Fairness (or Balanced contributions property — Myerson, 1980); it coincides with the Shapley value if g is a complete graph
- Recently, Gonzales-Aranguena e.a. (2008) have extended the Myerson value to directed graphs (digraphs).

The process of strategic bargaining

Each offer and each voting changes the strategic situation.

Let X_i^t be the set of strategies of player i at time t.

Example: Suppose there is an S-game, and player 1 has made an offer to player 2. Then

- $X_1^1 = \{$ wait till another offer (W), offer a different bid (O), wait if 2 accepts (E)$\}$
- $X_2^1 = \{$ wait till another offer (W), offer a different bid (O), accept 1's offer (A)$\}$
- $X_3^1 = \{$ wait till another offer (W), offer a different bid (O)$\}$

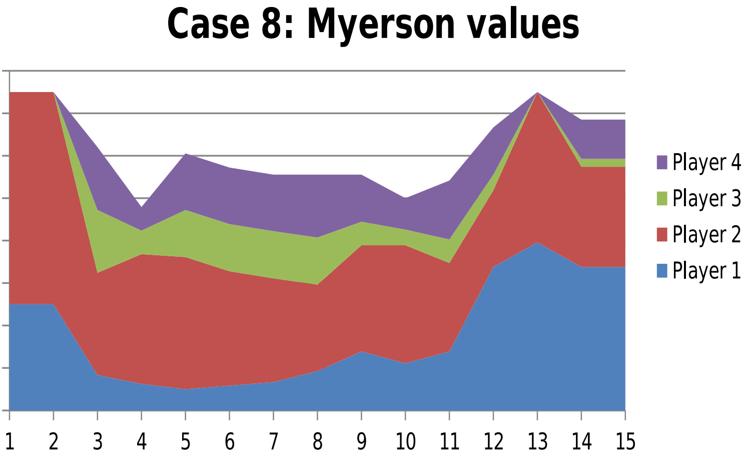
Assuming all offers are being made simultaneously, each player anticipates random evolution of the graph, and chooses

$$x_i^{t*} = \arg\max_{x_i^t \in X_i^t} u(x_i^t, \mathrm{E}x_{-i}^{t*}), \forall i, j, t$$

at each stage of the game, altering the Myerson value at each stage.

Dynamics of the Myerson values

F-game, bargaining path



Strategic Approach to Nonstructured Bargaining II

Consider a general simple cooperative game (S, v) over the set of all possible game states X (incorporating future possible states and beliefs of players about these).
Assumption 1: The set of states is locally compact
Assumption 2: The improvement correspondence $\Gamma \equiv \times_{i=1}^{N} \mu_i : X \to [0,1]$ (is the product correspondence of possible dynamic Myerson values) is upper semicontinuous mapping from the set X onto itself
Proposition: If ΓX is nonempty and convex, there exists a fixed point in X, which is the set of equilibrium states.
Uniqueness: to be developed.

Conclusions

- Explicit modifiers work in all treatments of the S–1 games, and increase the payoff of player 3 by about 21%. Effects for the other players are not significant.
- Implicit modifiers in the S–games can be suppressed by centering the players and other means.
- Explicit modifiers (probably) do not work in the V–2 games.
- The intensity of connections of other players to the given player *i* (in some contexts) matters more for her payoff.
- Explicit modifiers work in the enlarged treatments as well.
  - Negative modifiers significantly affect the frequency of the respective coalitions in the E–games: players tend to switch to a larger coalition comprising the players with neutral modifiers.
- Modifiers of opposite nature interact in a complex manner.
- Predictive power of the classical power indices is ambiguous: the best explanatory variables are player role and winning coalitions.

Q & A

The latest version of the paper can be downloaded from
http://epee.hse.ru/project

(the website of the Laboratory for Behavioral and Experimental Economics, State University - Higher School of Economics)

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