The Role of (Negative) Emotions in Self-Control

(Very preliminary and incomplete draft)

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Abstract

We study a dynamic model of emotional self-control where the history of one's self control decisions (understood as emotions) has influence on subsequent decision making. In our approach, effort and regret are emotional consequences of decisions to resist or succumb to temptation that can affect current payoffs. We describe results of non-stationary consumption paths characterized by compensatory feasting and fasting cycles, the effects of emotional memory and foresight, a last period craving effect in finite time series, and the effects of unexpected temptations and menu choices.

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"The only way to get rid of a temptation is to yield to it. Resist it, and your soul grows sick with longing for the things it has forbidden to itself" (Oscar Wilde, The Picture of Dorian Gray, 1891)

"The dieter spots the German chocolate cake on the dessert trolley and is sorely tempted. His rational assessment urges restraint. The weight he will gain, it tells him, is too big a price to pay for the fleeting moment of gastronomic pleasure. Yet, despite this calculation, he often gives in. And when he does, he almost invariably expresses regret." (Robert H. Frank, Passions within Reasons, 1988, p. 86)

1 Introduction

Temptation and self-control problems are among the most basic problems that humans struggle with in their moment-to-moment and day-to-day decisions. Many of us repeatedly roller-coaster through the same difficult cycles of indulgence and restraint time and again, sometimes feeling it is impossible to not avoid old patterns and to find a stable behavior. Despite the central importance of these fundamental problems, the system producing their dynamics has been poorly understood and remains among the most intriguing puzzles for economist and cognitive scientist to solve (e.g., see Brocas & Carrillo (2003); Tversky & Shafir (1992); Pinker (1997)).

The self-control problem presents an apparent paradox: standard economic theories are based on the assumption that consumers choose what they prefer, yet when self-control problems are faced we have evidence that despite stated intentions or preferences people often demonstrate difficulty "committing" to pursuit of those preferences (Strotz (1955)). A popular approach to formalizing this kind of behavior considers individuals' choices to be the products of two utility functions (or goal systems) with conflicting interests. As Schelling (1978) described, sometimes a person "behaves like two people, one who wants clean lungs and long life and another who adores tobacco, or one who wants a lean body and another who wants dessert. The two are in a continual contest for control".

The internal conflict idea has also been considered and justified in other disciplines. *Evolutionary psychologists* (Tooby et al. (2008)) propose adaptive design features of internally conflicted systems (composed of mutually exclusive goal pursuant components) perhaps as adaptive solutions to changing environments (Tooby & Cosmides (1992)). Moreover, recent developments in *neuroscience* seem to confirm the idea of a brain composed of conflicting modules. For instance, using brain imaging studies McClure et al. (2004) show that two separate neural subsystems are activated when self-control problems are faced and that the relative engagement of one system over the other determines how future utility is traded-off against present utility. Moreover, Livnat & Pippenger's (2006) analysis of decision making systems finds that under limited computational ability an optimal brain system may involve subsystems that are each in conflict with each other, and despite natural selection acting on whole systems, such internally conflicted systems still manage to evolve.

An individual characterized by two utilities with conflicting goals cannot maximize both preferences at the same time. Therefore, each decision creates unfulfilled preferences or "forgone utilities" -the difference between the maximum possible utility and the realized utility. The purpose of this paper is to introduce the effects of these forgone utilities (emotions) on future preferences and decisions.

Before explaining the key elements of our model we briefly described Gul and Pesendorfer (2001) (here forward GP) popular self-control representation of precommitment behavior and its limitations. GP propose an individual with two different utilities: u, the normative utility, and v, the temptation utility, who suffers self-control costs when temptation preference is not fully satisfied. This model of consumer decision can be easily interpreted in two stages. In stage 1, the individual chooses a menu (i.e., a set of available alternatives). In stage 2, a consumption choice is made from among alternatives on the menu. Note that for the standard single utility individual there is no benefit of parsing choice into two stages: in stage 1 the menu containing the most preferred alternative would be selected and in stage 2 that most preferred alternative would be chosen for consumption. Furthermore, the single utility individual is never worse off with a broader menu (preference for flexibility). However, in GP's self-control representation, behavior can be different than that of a single-utility individual. In the second stage, once the menu has been chosen, the individual makes consumption choices from available alternatives that maximize both u and v. In striking a compromise between the conflicting utilities GP's individuals incur a "cost of self-control". This is a consequence of the forgone temptation utility, the difference between the maximum value of v over the menu and the realized value of v. In particular, if $\widehat{x} \in M$ is the realized choice, the individual overall utility from menu M is given by

$$
\begin{aligned}
W^{GP}(M) &= \max_{x\in M}\left[u(x)+v(x)\right]-\max_{x\in M} v(x) = \\
&= u(\widehat{x})-\left[\max_{a\in M} v(x)-v(\widehat{x})\right].
\end{aligned}
$$

Therefore, the overall utility from a choice set equals the normative utility minus the self control cost. The self-control costs make it clear that a larger menu can be less desirable since it may yield a higher value of $\max_{x\in M} v(x)$. Thus, the individual presents *preferences for commitment* since he prefers a subset of the menu to the menu itself.[1] However, consumption choices made from menus are unaffected by previous self-control costs since, once the menu has been chosen, the consumer chooses from among available alternatives so as to maximize normative and temptation preferences (i.e., $\widehat{x} \in \arg\max_{x\in M}\left[u(x)+v(x)\right]$). Note that the actual choice depends on the relative cardinal scale of the utilities which represent the bargaining power of u and v. For example, if the relative power of v is smaller, consumer's choice will be more powerfully influenced by the normative preference.[2]

The previous self-control representation implies that, if we consider a dynamic model with a given menu, consumer choice paths will be stationary. This is, however, not consistent with observed behaviors. For instance, psychiatric and health-care professionals have documented pathological patterns (e.g., anorexia and bulimia) cycling between periods of binge consumption and compensatory fasting (Kurth et al. (1995); Heatherton et al. (1995))). Moreover, psychologist have pointed out that these symptoms are closely related with and regulated by negative emotional experiences. In particular, as a consequence of these extreme consumption behaviors, individuals report high negative affect after bingeing and purging (Lynch et al. (2000)).[3] Casper & Beaton (1992) describe analyses of data from the Beltsville one year dietary study (Mertz & Kelsay (1984)) showing

[1] GP refer to u as the *commitment utility* because it represents the agent's ranking of singleton (i.e., full commitment) menus. Since the main focus of this paper is not on preferences over menus but on choices from menus, when referring to u we prefer the terms *normative utility* used by Noor and Takeoka (2011) or Kopylov (2012), or else "long-sighted utility" used by economists and psychologists (Carrillo (1998); Logue (1988); Rachlin (1995)). When referring to v we prefer either terms "temptation utility" or else "short-sighted utility".

[2] See literature review where describe generalizations of self-control preferences. Also see Lipman & Pesendorfer (2011) for a more exhaustive literature review of the topic.

[3] These episodes could become pathological (bulimia nervosa) when individuals fail to regulate them (Whiteside et al. (2006)).

"normal" day-to-day caloric consumption patterns in non-bulimic populations that cycle between positive and negative energy balance. Similarly, experts in marketing have found that when decision makers with conflicting preferences recall past behaviors, they tend to inhibit fulfilled preferences and activate unfulfilled ones (Forster et al. (2005)).[4] Based on this idea, Mukhopadhyay et al. (2008) found that recalling past behaviors of resisting or succumbing to the temptation of tasty but unhealthy food leads to a greater activation of unfulfilled preferences. Thus, impulsive consumers tend to do the opposite of what they have done in the past: resisting if they recall having succumbed to temptation and succumbing to temptation if they recall having resisted. Similar patterns, like those seen with *feasting and fasting behaviors*, are often observed in everyday activities such as shopping, working, moral behaviors, sex, and leisure activities.

In the present paper we introduce a self-control model that integrates emotional costs according to the history of previous self-control decisions and provides explanations for the non-stationary behaviors that individuals produce as they make consumption choices according to menu options. In particular, we consider that previous forgone utilities (experienced as emotions) could affect present decisions through their impact on preferences. If we called e_t the history of forgone temptation utilities and g_t the history of forgone normative utilities, the individual's (overall) utility in period t is given by

$$W_t(M) = \max_{x \in M} \left[U(x, e_t) + V(x, g_t) + \delta W_{t+1} \right]$$

with

$$\begin{aligned} e_{t+1} &= (1-\lambda) e_t + \left[\max_{x \in M} v(x) - v(x_t) \right], \\ g_{t+1} &= (1-\lambda) g_t + \left[\max_{x \in M} u(x) - u(x_t) \right]. \end{aligned}$$

Where $(1-\lambda)$ represents the degree at which forgone utilities are recalled.

The main difference between the above and a standard self-control representation is that in the above case, forgone utilities (captured by e_t and g_t) affect the bargaining power between the individual's preferences and hence they will have an impact not only on menu choices but also on choices from menus. In other words, normative and temptation preferences (i.e., long-sighted and short sighted goals) are defined by not only the current actions but also the stock of previous self-control decisions represented by the emotions.

[4] Marketing experts and psychologists typically refer to conflicting preferences as *conflicting goals.*

We interpret e_t as an effort-like emotion that is experienced as a consequence of resisting temptation (and forgoing the short-sighted goal of indulging temptation) and g_t as a regret-like emotion that is experienced as a consequence of succumbing to temptation (and forgoing the longsighted goal of temperance). Both emotions are negative in the sense that they impose costs, decreasing the cardinal scale of the targeted utilities. Effort targets u imposing cost to down-regulate its bargaining power while regret targets v imposing cost to down-regulate its bargaining power over the consumption decision.

To facilitate the analysis and interpretation of our model we use a Hotelling specification where mutually exclusive and conflicting preferences are located at the extremes of a unit length segment. This "Hotelling line" provides an intuitive way to capture differences in normative and temptation preferences as different "ideal points" in a characteristic space. For example, imagine an individual who wants to consume healthy, low calorie foods according to his normative preference (because it is more beneficial according to long-term goals like health-maintenance) but also wants to consume tasty, high-calorie foods according to temptation preferences (because it immediately satisfies short-term goals like craving-satiation). A menu, that represents the alternatives faced by the individual, will be a subset of all possible alternatives included in the Hotelling line. Therefore, under this specification, forgone utilities (and hence negative emotions) are proportional to Euclidean distances between the point of an actual consumption alternative (i.e., menu item) and the boundaries of the menu. If the individual has been taking actions closer to the normative ideal point, the "effort" experience from resisting temptation will be higher, in turn decreasing the relative weight of normative preference. Thus, future attempts to resist temptation will be more difficult. Similarly, if the individual has been taking actions closer to the temptation ideal point, "regret" experienced from succumbing to temptation will be higher, in turn decreasing the relative cardinal scale of temptation preference. Thus, future attempts to resist temptation will be easier.

We find several results providing the non-stationary evidence of self-control behaviors. First, we consider a model with a menu that is given and perfectly known by the individual. We show that the optimal consumption path follows compensatory feasting and fasting cycles and a last period craving effect whose shape and amplitude depend on the given menu (M), the individual discount factor (δ) and his emotional memory ($1 - \lambda$). Second, we consider a model where, at the beginning of the game, the individual decides on a *commitment menu* that he will face in subsequent consumption periods unless uncontrolled alternatives arise (which we examine the consequences of). We can interpret this model by considering the consumer who shops for his week's meals at the beginning of the week (commitment menu) but later encounters unexpected alternatives (commercials, products in the supermarket shelves, etc.) affecting his set of available alternatives at the time of consumption. As we show, our model predicts that the individual demands a commitment menu with some (but not full) flexibility in order to adapt his consumption to the emotions created by the uncontrolled alternatives. These results lie in between the those predicted by standard utility models (preference for flexibility) and self-control models with linear self-control costs (preference for commitment).

Literature Review

There are at least two important economic literatures that are related to our work: one on addiction and another on temptation and self-control.

In the literature on *addiction* our paper relates most to models that account for an extended utility function (Becker (1996)). These models provide a function defined, not only by present consumption trade-offs but also by the stock of past consumption, called *consumption capital*. The important novel feature of this class of models is that while preferences remain stable, they are defined by not only "ordinary goods" but also by features (e.g., past consumption, culture or emotions) not normally thought of as "goods". For instance, Becker and Murphy (1988) provide a theory of rational addiction where present marginal utility increases with consumption capital. Thus, consumption of an addictive good presents "adjacent complementarity" across periods. In this model, addiction is rational in the sense that the individual is forward looking and preferences are stable. Thus, in contrast to a dual-utility model, precommitment behavior is never optimal because self-control is not costly for the individual. In other words, a rational addict would quit whenever he consider quitting to yield greater benefits (taking into account present and discounted future benefits) than continuing with the consumption of the addictive good. Laibson (2001) extends this "rational addiction" idea to include the effects of environmental cues which triggers consumption of an addictive good. Bernheim and Rangel (2004) also consider a model with environmental cues. However, in their model addiction can be irrational in the sense that a cue generates a temporary loss of control that leads to consumption of the addictive good independently of any negative future consequences.

In our model the extended utility concept is used to capture the non-stationary behavior that is so commonly found among those struggling with self-control problems. In light of previous theoretical work, our extended utility analysis includes several novelties. First, in considering an individual's two internally-conflicting preferences we specify two different extended utilities. More importantly, in determining the marginal utility of consumption we model consumption capital as captured by stocks of previous forgone utilities (and their emotional consequences), not just a single stock of previous consumption. Since forgone utilities depend upon options not chosen, the menu's set of alternatives (specifically, the available options) plays a very important role in determining extended utilities. Finally, our concept of negative emotions departs from previous models of addiction because emotionally regulated consumption presents "adjacent substitutability" across periods. Thus, in contrast to the standard consumption capital in models of addiction, in our model, marginal utilities decrease with the experience of negative emotions.

As we have just mentioned, GP provide a popular representation of *temptation and self-control* that uses a dual-utility model to capture the idea that self-control can be costly and incomplete. There are several extensions of the GP preferences. Kopylov's (2012) extension of GP preferences also incorporates forgone u-utility, demonstrating that options maximizing normative preference can also be costly when choosing menus. In Kopylov (2012) forgone u-utility can also be valued by consumers. The author provides the perfectionist result of an individual who values a menu with greatest normative options but least tempting options, even if he fails to take advantage of the available normative options later. This behavior is consistent with the findings of DellaVigna and Malmendier (2006) about "paying not to go to the gym", that people enroll in health-club memberships that are not taken advantage of. Sarver (2008) provides another model incorporating the forgone normative utility which he calls regret. Unlike our model where regret also affects choices from menus, Kopylov's (2012) and Sarver's (2008) regret only affects choices of menus.

Some authors have criticized GP´s idea of linear self-control costs and its implications for menu preferences. Dekel et al. (2010) provides a contrasting view to the GP assumption that only the most tempting offer determines self-control costs. In the Dekel et al. (2010) representation, self-control costs can be affected by several options on the menu. Fudenberg and Levine (2006, 2010) consider a version of the GP representation where the self-control cost is nonlinear. In these models more numerous temptations are more difficult to resist because of cognitive load that is thought to increase with each additional temptation. In other words, marginal self-control cost increases with the number of available options on the menu. Noor and Takeoka (2011) consider yet another extension of GP´s representation where cost of self-control is the forgone temptation utility scaled up by the "degree of temptation" as represented by the most tempting offer on the menu. Hence, a higher degree of temptation increases the weight on temptation utility. Therefore, as in our model, Noor and Takeoka (2011) also consider a model with menu-dependent self-control. However, in contrast with their paper, we consider a self-control model where the menu not only affects temptation but also normative preference. Moreover, in our representation self-control is affected by previous forgone utilities; not only affecting choices of menus but also choices from menus. Therefore, a higher value of the most tempting offer (i.e., a higher "degree of temptation") will increases the cardinal scale of temptation preference, as long the effort produced by a choice from the menu is greater than the regret produced. But, as we shall see, in our model this is not always the case.

A few papers have introduced time in a self-control model. Gul and Pesendorfer (2004) provide axioms for a recursive self-control model where, each period, an individual makes a consumption decision and chooses a menu for the next period. Using this recursive self-control idea, Gul and Pesendorfer (2007) study a model where the individual can consume two kinds of goods: a normal good and an addictive good or drug. Although the normative utility can depend on both kinds of goods, the temptation utility only depends on the consumption of the addictive good. Addiction is introduced in the model because previous consumption of the addictive good determines the marginal temptation utility in the next period (that they call the state). The authors apply this class of preferences to explain the case where a consumer faces a constant menu over time but can decide to enter to a rehabilitation center where drug consumption is forced to zero. The authors show that, in equilibrium, the consumer increases drug consumption until deciding to enter rehab. Explained in terms of internal conflict: as drug consumption increases, the "bargaining power" of the tempted self decreases until the bargaining power of the normative self prevails: leading the individual to check into rehab and remain until released, at which point the addiction/rehab process starts again. To our knowledge, Gul and Pesendorfer (2007) is the only paper that incorporates the idea of addiction, and hence nonstationary behavior, in a dual-utility self-control model, making it the paper most closely related to our work. However, our analysis has many important differences. First, we consider emotional capital (i.e., an extended utility function): consumption today depends on consumption history, not only on the last period. More importantly, in our model, the production of emotions comes from the difference between the maximum utility that the selves can get and the utility that they really get. Therefore, the "state" not only depends on previous consumption but also on the set of available alternatives. Moreover, in contrast with Gul and Pesendorfer (2007) our notion of emotions also incorporates regret (a consequence of forgone normative utility). As we shall see, internal regulation of the conflicting preferences is managed by these two emotions. Finally, one of our model´s novel contributions derives from its ability to consider various effects of uncontrolled options in menus which can have disruptive effects on an individual´s state. An important result derived from consideration of the hazards is that rational individuals commit to flexible (not highly constrained) menus allowing them to adapt consumption to changes in the emotional state created by the uncontrolled options that can arise in the menu (for instance, as a result of products advertisements).

In sum, the key analytical contribution of our dual-utility self-control model with negative emotions extend from its ability to consider emotions, the history of forgone utilities (i.e., in terms of the interplay between preferences, historical decisions and menu of options), as the determinants of an individual's self-control.

2 The Model

We consider an individual deciding on actions over several periods of time. Time is discrete and indexed by $t = 1, 2, ..., T$, where $T < \infty$. Individual's actions are represented as locations on a Hotelling line of unit length $[0, 1]$. We denote by x_t the individual's action (or consumption choice) in period t and define M as the time invariant compact set of available alternatives (or menu).[5] The menu is a closed interval that lies in the Hotelling line. Thus, $M \equiv [\underline{m}, \overline{m}] \subseteq [0, 1]$ with $\underline{m} \leq \overline{m}$. We assume that the individual is perfectly informed about the alternatives in the menu.

Consistent with dual-preferences models in behavioral economics and psychology we assume that the individual is characterized by two different utilities. We denote these utility functions by u, the normative utility, and v, the temptation utility. We assume that preferences are located at the extremes of the Hotelling line. Without loss of generality we consider that the normative ideal point is located at zero while the temptation ideal point is located at one. Therefore, the individual utilities are given by:

$$\begin{aligned} u(x_t) &= s - t(x_t, 0), \\ v(x_t) &= s - t(x_t, 1). \end{aligned}$$

[5]In Section 5 we will also consider the case of a time-variant set of alternatives.

where $s \in \mathbb{R}_+$ represents the maximum surplus and $t(x, \theta) = (\theta - x)^2$ with $\theta \in \{0, 1\}$ represents the transportation costs.

As discussed in the introduction, when an individual with mutually exclusive dual-preferences makes a decision he cannot maximize both utilities simultaneously. Given the individual action x_t in period t, the forgone utilities of each competing preferences u and v are $i^u = \max_{x \in M} u(x) - u(x_t)$ and $i^v = \max_{x \in M} v(x) - v(x_t)$ respectively. Note that *resisting temptation,* i.e., x_t close to zero, creates a high forgone $v-$utility (but a low forgone $u-$utility), while *succumbing to temptation*, i.e. x_t close to one, creates a high forgone $u-$utility (but a low forgone $v-$utility). We define *emotional capital* as the sum of all previous forgone utilities depreciated at a constant rate. In particular, the *effort capital* transition equation is given by

$$e_{t+1} = (1 - \lambda) e_t + \left[\max_{x \in M} v(x) - v(x_t) \right],$$

while the *regret capital* transition equation is

$$g_{t+1} = (1 - \lambda) g_t + \left[\max_{x \in M} u(x) - u(x_t) \right].$$

Where initial conditions $e_1 \in \mathbb{R}_+$, $g_1 \in \mathbb{R}_+$, and $\lambda \in [0, 1]$ represents the psychological depreciation rate of emotions. We refer to $(1 - \lambda)$ as *emotional memory.* Thus, under this interpretation emotional capitals are the remembered forgone utilities at a particular point in time while forgone utilities, i^u and i^v, are the new emotions created every period, that is, the production of (or investment in) emotional capital. Moreover, we define an individual's *emotional balance* as the difference between the two kinds of emotional capitals:

$$B_{t+1} = e_{t+1} - g_{t+1} = (1 - \lambda) B_t + 1 - 2x_t + \mu(M)$$

where $\mu(M) = \underline{m}^2 - (1 - \overline{m})^2 \in [-1, 1]$ is a function that depends on boundaries of the menu. This function indicates that emotions depend, not only on previous actions, but also on available menu options. If $\mu(M) > 0$ the menu contains more alternatives closer to the temptation preference, in this case we can say that the menu is *temptation shifted*. Similarly, if $\mu(M) < 0$ the menu contains more alternatives closer to normative preference, in this case we say that the menus is *normative shifted.* Finally, if $\mu(M) = 0$ the menu does not favor one kind of preferences over the other, we can say that the menu is *neutral.* By notational convenience we will sometimes refer to this function just as μ.

Now we can define the following *extended utility* functions in period t that depend on the actual action (the state variable) and the emotional capital (the control variable) at each date:

$$U(x_t, e_t) = u(x_t) - \rho\varepsilon(x_t, e_t),$$
$$V(x_t, g_t) = v(x_t) - \rho\gamma(x_t, g_t).$$

Where $\rho \in \mathbb{R}_+$ and $\varepsilon(x_t, e_t)$ and $\gamma(x_t, g_t)$ are emotional cost functions. In particular, we consider that $\varepsilon(x_t, e_t) = e_t(1 - x_t)$ and $\gamma(x_t, g_t) = g_t x_t$. These functions capture the idea that decisions can be emotionally costly. The closer x_t is to the temptation ideal point, $\theta = 0$, the higher the individual's emotional cost of effort. While the closer x_t is to the normative ideal point, $\theta = 1$, the higher the individual's emotional cost of regret. The linearity of the functions assures that the marginal costs of the individual's decision are given by the emotional capitals. Thus, to succumb to temptation is costly (taking the form of regret costs) and it is even more costly if the individual has previously succumbed to temptation, and hence accumulated regret capital. Similarly, resisting temptation is costly (taking the form of effort cost) and more costly if the individual has previously resisted temptation, and hence accumulated effort capital.[6]

Note that, while the primitive utility functions, u and v, are constant, the extended utilities that incorporate emotional capital depend on t and are therefore *non-stationary.*

We introduce an *emotional self-control representation* that incorporates our notion of costly emotions.

$$W_t(M) = \max_{x \in M}\left[U(x, e_t) + V(x, g_t) + \delta W_{t+1}\right].$$

Where $\delta \in [0, 1]$ is the individuals discount factor. Note that if $\rho = 0$ we have a standard model with no history effects where individual preferences are $W(x_t) = u(x_t) + v(x_t)$. However, the key point of our representation arises when $\rho > 0$, that is, when the emotional capitals affect the relative cardinal scale of the extended utilities. In this case previous decisions to resist or succumb to temptation affect present decisions through emotional costs.

The action at date t will depend on the current emotions, $x_t = \alpha_t(e_t, g_t)$ where α_t is referred to as the decision rule. An optimal decision rule solves the following maximization problem every period:

$$W_t(M) = \max_{x \in M} U(x, e_t) + V(x, g_t) + \delta W_{t+1}.$$

[6] Alternatively, we can interpret these functions as "transportation costs" that act in the opposite direction of the standard transportation costs and whose magnitude increase with emotional capital (i.e., with previous self-control decisions).

Where e_t and g_t evolve according to the transition equations previously described. A decision (consumption) path is a sequence of decision rules $(\alpha_0, \alpha_1, ..., \alpha_T)$. We say that a decision path is *stationary* if decision rules do not depend upon time $\alpha_t(e,g) \equiv \alpha(e,g)$.

Note that, in contrast to a standard utility model ($\rho = 0$) and self-control models with linear self-control costs (GP or Kopylov (2012)), our representation implies a violation of the Weak Axiom of Revealed Preferences. To see this, let's consider $M = \left[0, \frac{1}{2}\right]$ and an individual with $\rho > 0$ who starts with no emotions (*i.e.*, $U(x_1, e_1) = u(x_1)$; $V(x_1, g_1) = v(x_1)$) and who does no discount the future (*i.e.*, $\delta = 0$). The action taken by this individual in the first period is $x_1 = \underset{x \in \left[0, \frac{1}{2}\right]}{\arg\max} u(x_1) + v(x_1) = \frac{1}{2}$. Therefore, the effort capital in period 2 will be zero while the regret capital will be $\frac{1}{4}$. This leads to the following extended preferences in period 2 :

$$
\begin{aligned}
U(x_2, e_2) &= u(x_2) \text{ and} \\
V(x_2, g_2) &= v(x_2) - \rho \frac{1}{4} x_2.
\end{aligned}
$$

Therefore, the action in period 2 will be $x_2 = \underset{x \in \left[0, \frac{1}{2}\right]}{\arg\max} U(x, e_2) + V(x, g_2) = \frac{1}{2} - \frac{\rho}{16} < \frac{1}{2}$. Thus, although the individual reveals a preference for $\frac{1}{2}$ over $\frac{1}{2} - \frac{\rho}{16}$ in period 1, he reveals the opposite in period 2. This is not the case when emotions plays no role ($\rho = 0$). In contrast with a standard self-control representation decision paths can be non-stationary in our model.

Although in this paper we mainly focus on choices from menus, and the implications for non-stationary behavior, we also consider choices of menus in section 5.

3 The Fully Myopic Benchmark

We find it useful to start our model analysis by considering the benchmark produced by a fully myopic individual: one who has no future thought and only preferences for the present ($\delta = 0$). This consumer's actions only take into account his present emotional capital, not how his decisions affect future emotional capital and hence future extended utilities and decisions. In this case, consumption decisions maximizes the sum of the two competing extended utilities $U(x, e_t) + V(x, g_t)$ every period. Therefore, $x_t = \frac{1}{2} + \frac{\rho}{4} B_t$. Hence, the consumer´s action is closer to the normative preference if the emotional balance is negative ($e_t < g_t$) and closer to the temptation preferences if the emotional balance is positive ($e_t > g_t$).

The next lemma summarizes the result of the myopic benchmark.

Lemma 1 *If the individual is myopic ($\delta = 0$) his consumption in period $t \in \{1, 2, ..., T\}$ is*

$$x_t = \frac{1}{2} + \frac{\rho}{4} B_t,$$

with $B_t = B_1 \left[1 - \lambda - \frac{\rho}{2}\right]^{t-1} + 2\mu \frac{1 - \left[1-\lambda-\frac{\rho}{2}\right]^{t-1}}{2\lambda + \rho}$.

In the following corollary we show under which circumstances the consumption path tends to a stable steady state.

Corollary 1 *If the individual is myopic ($\delta = 0$) and $\rho + 2\lambda < 4$ then there exists a stable steady state given by*

$$\lim_{t\to\infty} x_t = \begin{cases} \overline{m} & \text{if} \quad \frac{\mu\rho}{2\lambda+\rho} \geq 2\overline{m} - 1, \\ \frac{1}{2}\left(1 + \frac{\mu\rho}{2\lambda+\rho}\right) & \text{if} \quad \frac{\mu\rho}{2\lambda+\rho} \in \left(2\underline{m} - 1, 2\overline{m} - 1\right), \\ \underline{m} & \text{if} \quad \frac{\mu\rho}{2\lambda+\rho} \leq 2\underline{m} - 1. \end{cases} .$$

Graphically,

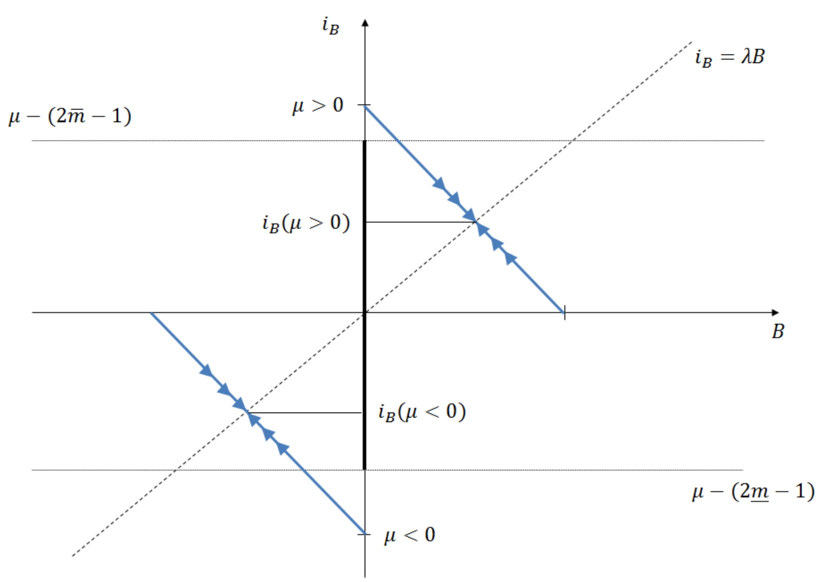

Figure 2.1. The steady state in the fully myopic benchmark.

The horizontal axis in Figure 2.1 graphs the emotional balance, while the vertical axis graphs investments in emotional balance. Thus, $i_B = \left[\max_{x \in M} v(x) - v(x_t)\right] - \left[\max_{x \in M} u(x) - u(x_t)\right] = 1 - 2x_t + \mu$. The blue lines represent the amount an individual invests in emotional balance as a function of the emotional balance $\left(i_B = \mu - \frac{\rho}{2}B\right)$, the vertical lines $\mu - (2\underline{m} - 1)$ and $\mu - (2\overline{m} - 1)$ represent the bounds of the menu. If $\mu > 0$ an interior steady state has a positive emotional balance (effort greater than regret) while if $\mu < 0$ the emotional balance will be negative (regret greater than effort).

By Corollary 1 we know that a fully myopic individual action tends to a stable steady state when emotions have a small impact on the individual preferences (ρ is low) or when the individual remembers most of the previous emotions (λ is low). Note that, although equivalent in the individuals actions, both cases have different implications for individual's payoffs. In the first case, the individual just doesn't care much about negative emotions so regret and effort don't impose significant costs and don´t have a big impact on decisions. However, in the second case the individual's recall of past emotions, although costly, calibrate consumption and achieve consistent behavior.

In figure 2.2 we plot the decision path of the myopic individual.

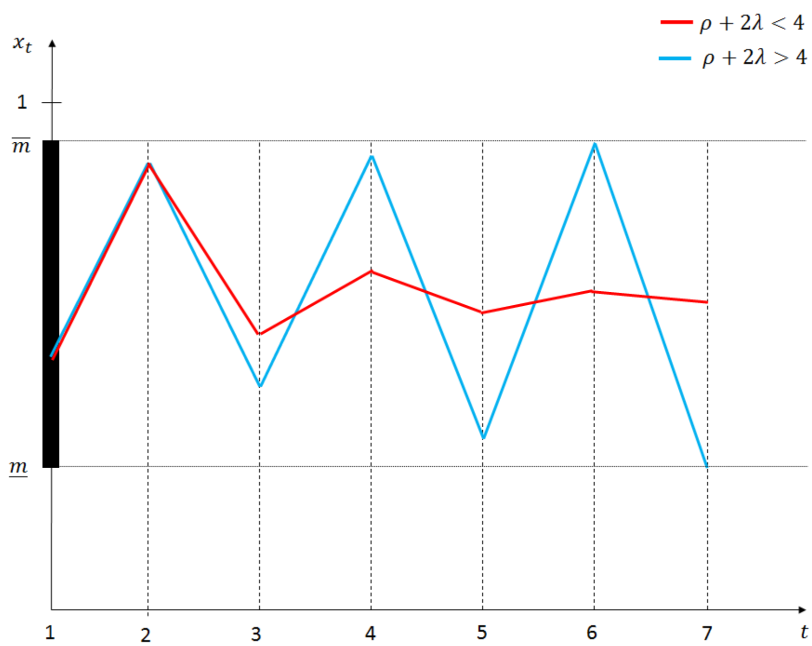

Figure 2.2. Decision paths in the fully myopic benchmark: stability and instablity.

Note that the decision path follows compensatory feasting and fasting cycles where the individual alternates between periods of succumbing to temptation, and hence high regret, with periods of resisting temptation, and hence high effort. If the impact of emotions (ρ) and the depreciation rate (λ) are sufficiently high, this behavior persists indefinitely. Otherwise, the amplitude of the cycles decreases over time and consumption tends to a stable steady state.

4 Characterization of the Optimal Consumption Path

In this section we compute the optimal decision path for any $\delta \in [0,1]$. We know that consumption in the last period is given by $x_T = \frac{1}{2} + \frac{\rho}{4} B_T$. Where B_T is the consequence of previous self-control decisions. Moving backwards and solving $\max_{x \in M} U(x, e_t) + V(x, g_t) + \delta W_{t+1}$ recursively we get the following result summarized in Lemma 2.

Lemma 2 *Let's consider $\rho \in [0,2]$. Solving the problem recursively we get that for all $t < T$:*

$$
\begin{aligned}
x_1 &= \beta_1 x_2 + \gamma_1, \\
x_t &= \alpha_t \left[x_{t-1} + (1-\lambda) x_{t-2} + ... + (1-\lambda)^{t-2} x_1 \right] + \beta_t x_{t+1} + \gamma_t \text{ for all } t \in \{2, ..., T-2\} \text{ and} \\
x_{T-1} &= \alpha_{T-1} \left[x_{T-2} + (1-\lambda) x_{T-3} + ... + (1-\lambda)^{T-3} x_1 \right] + \gamma_{T-1}.
\end{aligned}
$$

Where α_t, β_t and γ_t are recursive functions defined in the Appendix.

The function contained in Lemma 2 is not a solution in itself but a relation between optimal decisions in different periods: Today´s action (x_t) is a linear function of past decisions ($x_1, ..., x_{t-1}$) and tomorrow´s action (x_{t+1}). However, note that we can plug $x_1(x_2)$ in $x_2(x_1, x_3)$ and solve the resulting equation to get $x_2(x_3)$. If we keep doing this we get x_t as a function only of x_{t+1} :

$$x_t = \frac{k_t + \gamma_t + \beta_t x_{t+1}}{1 - \alpha_t \beta_{t-1} h_t} \text{ for all } t \in \{2, ..., T-2\} \tag{1}$$

where $h_t = \frac{1+\beta_{t-2}(1-\lambda)h_{t-1}}{1-\alpha_{t-1}\beta_{t-2}h_{t-1}}$ and $k_t = \alpha_t \left(h_t \left(k_{t-1} + \gamma_{t-1} \right) + (1-\lambda) \frac{k_{t-1}}{\alpha_{t-1}} \right)$.

Now we can use $x_{T-1} = \frac{k_{T-1}+\gamma_{T-1}}{1-\alpha_{T-1}\beta_{T-2}h_{T-2}}$, which is a known scalar, and find the optimal decision path following the transition equation (1).

To understand the optimal decision path we provide a comparative statics analysis based on simulations.[7] We consider that the initial emotional balance is $B_1 = 0$ and the menu is temptation shifted ($\mu > 0$).

In Figure 3.1. we can observe the effects of emotions on the consumption path. In red we have a model where emotions plays no role ($\rho = 0$) and in blue we have the case with emotions ($\rho > 0$). As we can see emotions create a *non-stationary consumption path*.

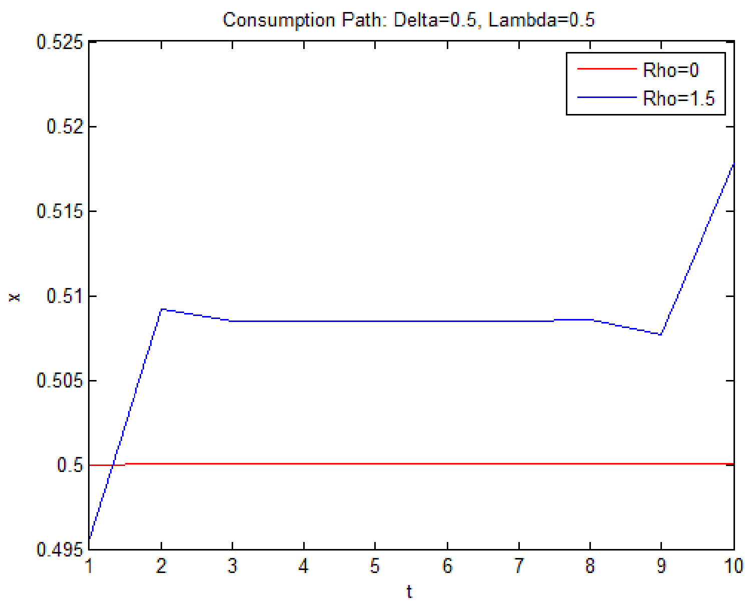

Fig. 3.1. Non-stationary consumption path.

This path is affected by the menu. When the menu is temptation shifted there are more options to resist close to temptation preference. Thus, the effort emotion has a comparatively larger effect in turn favoring the bargaining power of temptation preference such that consumption will tend to be closer to one. As we observe (see Figure 3.2.), if the menu is normative shifted ($\mu < 0$) we have the symmetric effect. Under normative shifted conditions the menu favors regret, which increases the bargaining power of normative preference, so consumption tends to be closer to zero. Whereas, if the menu is neutral ($\mu = 0$) both emotions are equally important and consumption is stationary.

[7] These simulations are the guesses of the general results that are under construction.

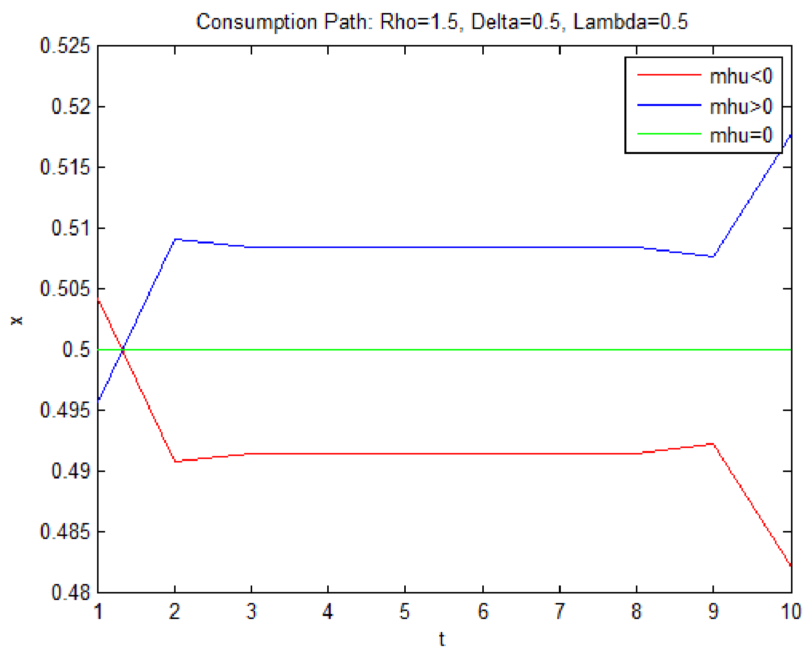

Fig. 3.2. Menu effects.

In the following figure we compare a myopic individual ($\delta = 0$) with a forward-looking individual ($\delta = 1$) who both have short emotional memories (i.e., except for the most recent ones, all previously produced emotions are forgotten).

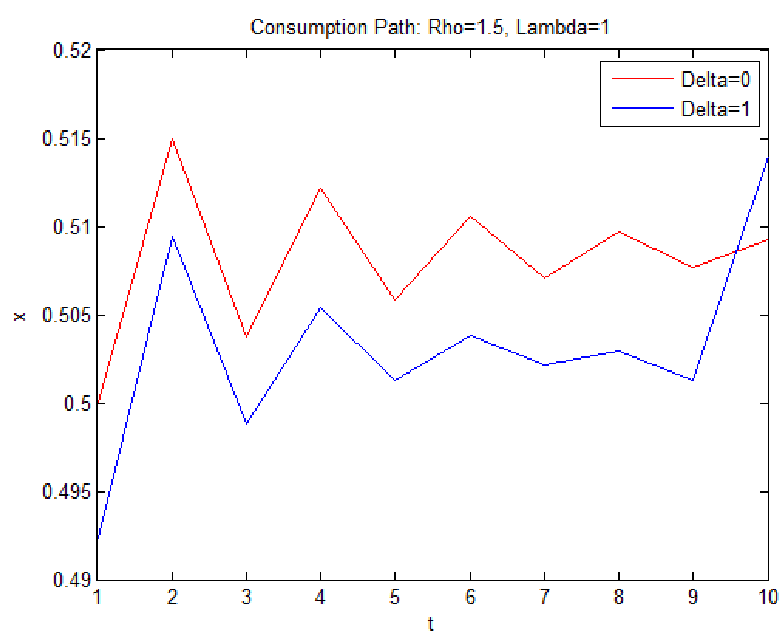

Fig. 3.2. Compensatory Feasting and Fasting Cycles.

We observe that the consumption path above follows *compensatory feasting and fasting cycles*. In the last section we obtained these cycles for a myopic individual, while here we check whether they also hold for a forward looking individual. We note an interesting effect: compared with a myopic individual, a forward looking individual consumes less in the first period but more in the last period. The underlying strategy employed by the forward-looking individual here is one that manages his emotions by keeping consumption low (across all periods but the last) so as to minimize the costs that would otherwise prevent him from succumbing to temptation in the last period. In other words, if the individual expects to succumb to temptation (increasing x) tomorrow he anticipates a high regret cost, $\gamma(x, g)$. Since the marginal cost of regret tomorrow increases with the regret (forgone normative utility) created today, he tries to keep today's consumption closer to normative preference (decreasing x) in order to keep tomorrow's regret low. This makes succumbing to temptation less costly tomorrow. Interestingly, the difference between consumption in the last and the first periods increases with δ, so in this sense, the more a consumer anticipates the future, the more inconsistency he will show with consumption.

Finally, in Figure 3.3 we compare a myopic individual ($\delta = 0$) with a forward-looking individual ($\delta = 1$) who both have long emotional memories (i.e., all previously produced emotions are recalled). In this case the *last period craving effect* for the forward-looking individual is even more intense.

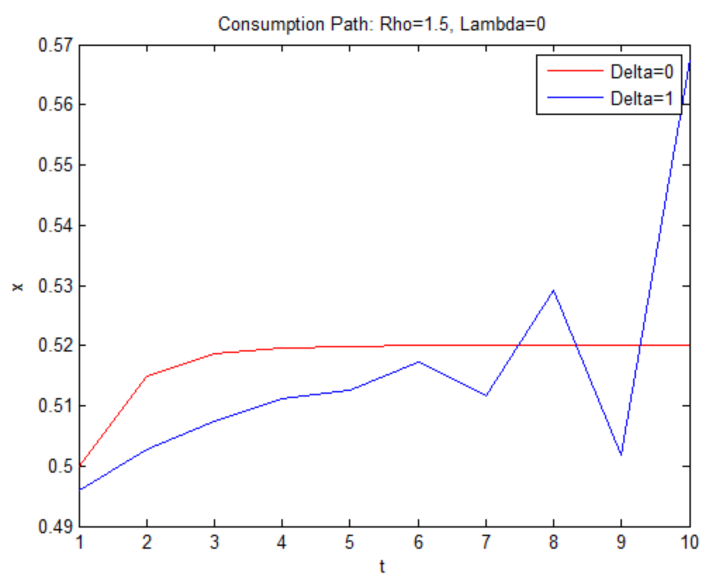

Fig. 3.3. Last period craving effect.

5 Menu Uncertainty and Commitment

In this section we relax the assumptions that the consumer has perfect information and no control over the alternatives in the menu. In particular we consider a three period, $t = \{0, 1, 2\}$, version of the model with initial emotions $e_1 = g_1 = 0$. In the first period $(t = 0)$ the individual has to decide the set of available alternatives that he is going to face in the future. At the beginning of $t = 1$ a set of uncontrolled alternatives realizes extension of the previously chosen set of available alternatives. Then, consumption takes place in $t = 1$ and $t = 2$. We refer to the set of alternatives that the individual chooses in $t = 0$ as the *commitment menu* -M- to the uncontrolled alternatives as *menu extension* -Z- and to the menu faced in $t = 1, 2$ as the *extended menu* -$\overline{M} = M \cup Z$. The menu extension is uncertain in $t = 0$, by simplicity we consider:

$$Z = \begin{cases} 1 & \text{with probability } \frac{p}{2}, \\ 0 & \text{with probability } \frac{p}{2}, \\ \phi & \text{with probability } (1-p)\,. \end{cases}$$

Therefore, $p > 0$ is the probability that new uncontrolled alternatives are created in $t = 2$.

By the symmetry of the extension around $\frac{1}{2}$, it is immediate that there is no loss of generality in looking at symmetric commitment menus: $M = [1 - m, m]$ with $m \in \left[\frac{1}{2}, 1\right]$.

We can interpret M as food that a consumer has purchased for meals and Z as commercials for alternative meals that the individual faces with probability p. The commercial can advertise an unhealthy product in line with temptation preference $(Z = 1)$ or a healthy product in line with normative preference $(Z = 0)$.

Before explaining the optimal menu choice resulting from our emotional self-control representation let's consider the menu choice in a standard utility model and in a self-control model with linear self-control costs. Under a standard utility representation where emotions play no role $(\rho = 0)$ the individual is indifferent to every menu containing $\arg\max_{x \in M} u(x) + v(x) = \frac{1}{2}$, the compromise between conflicting preferences. In other words a higher menu can only be better for the individual (preferences for flexibility). Moreover, as we already argued, a self-control representation with linear self control costs implies a stationary consumption path given a menu. Let's consider a Kopylov (2012) extension of

GP preferences that also includes forgone commitment utility in the model:

$$W^K(M) = \max_{x \in M} [u(x) + v(x)] - \max_{x \in M} v(x) - k \max_{x \in M} u(x)$$

where $k \in (0,1)$.

Note that $x_t = \arg\max_{x \in M} u(x) + v(x) = \frac{1}{2}$ for any M that includes the alternative $\frac{1}{2}$, the compromise between the individual's conflicting preferences. Since $k > 0$, any additional alternative $y \neq \frac{1}{2}$ creates negative consequences for the individual. If $y > \frac{1}{2}$, $\max_{x \in M} v(x) > v\left(\frac{1}{2}\right)$ when $Z \in \{\phi, 0\}$ while if $y < \frac{1}{2}$, $\max_{x \in M} u(x) > u\left(\frac{1}{2}\right)$ when $Z \in \{\phi, 1\}$, so consumer's expected utility in $t = 0$ decreases. In other words, by committing to the compromise between the conflicting preferences, the individual minimizes the negative consequences of the menu extensions and maximize overall utility, so the optimal menu will be $M^{FC} = \left\{\frac{1}{2}\right\}$ (preferences for commitment).

These above results are a consequence of the stationary of individual preferences. However, as we have seen, in our emotional self-control representation previous decisions can change the bargaining power of conflicting preferences creating a non-stationary consumption path.

In the next lemma we summarize individual consumption given the commitment menu chosen in $t = 0$.[8]

Lemma 3 *Given a commitment menu M, the optimal consumptions under each realization of Z are*

$$\begin{aligned} x_1\left(\overline{M}\right) &= \frac{1}{2} - \frac{\delta\rho^2}{8 + 4\delta\rho\left(1 - \frac{\rho}{2}\right)} \mu\left(\overline{M}\right), \\ x_2\left(\overline{M}\right) &= \frac{1}{2} + \frac{\rho(2 + \delta\rho)}{8 + 4\delta\rho\left(1 - \frac{\rho}{2}\right)} \mu\left(\overline{M}\right). \end{aligned}$$

with $\mu(M \cup \{1\}) = (1-m)^2$, $\mu(M \cup \{0\}) = -(1-m)^2$ *and* $\mu(M) = 0$.

Note that when the extended menu is temptation shifted $\mu\left(\overline{M}\right) > 0$, then $x_1 \leq \frac{1}{2}$ and $x_2 \geq \frac{1}{2}$. Thus, if the extended menu favors consumption close to temptation preference, the individual finds it optimal to consume closer to normative preferences in $t = 1$ in order

[8] To keep the analysis simple and facilitate the interpretation of the results, in this preliminary version of the paper we focus on interior solutions. Thus $x_t \in (1-m, m)$. An interior solution always exists for appropriate parameters, in particular when p is sufficiently high (See Lemma 4).

to keep regret low and the costs of succumbing to temptation low in $t = 2$. Similarly if the extended menu is normative shifted $\mu\left(\overline{M}\right) < 0$, then $x_1 \geq \frac{1}{2}$ and $x_2 \leq \frac{1}{2}$. Thus, if the extended menu favors consumption close to normative preference, the individual finds optimal to consume closer to temptation preference in $t = 1$ in order to keep effort low and make normative consumption less costly in $t = 2$. Finally, when the extended menu is neutral, $x_1 = x_2 = \frac{1}{2}$, it does not favor any particular emotion, so the emotional balance is zero (effort equals regret) and consumption is stationary.

Now we are ready to calculate the optimal commitment menu $M^* = [1 - m^*, m^*]$ which solves the following maximization problem faced by the individual in $t = 0$:

$$m^* \in \arg\max_{m \in \left[\frac{1}{2}, 1\right]} \frac{p}{2} W_1 \left(M \cup \{1\}\right) + \frac{p}{2} W_1 \left(M \cup \{0\}\right) + (1 - p) W_1 (M)$$

In the following lemma we summarize some properties of the optimal commitment menu

Lemma 4 *The optimal commitment menu is* $M^* = [1 - m^*, m^*]$ *where* m^* *satisfies:*

(i) $m^* > \frac{1}{2}$ *iff* $p > \overline{p}(\delta, \rho)$ *and*

(ii) If $m^* > \frac{1}{2}$ *then* $\frac{\partial m^*}{\delta p} > 0$, $\frac{\partial m^*}{\delta \delta} > 0$ *and* $\frac{\partial m^*}{\delta \rho} > 0$.

The first property says that an individual with emotional self-control problems may want to commit to a menu with some (but not necessarily full) flexibility. This is an important novel result that offers new insights for the "preference for commitment" *vs* "preference for flexibility" debate. In our model, the uncontrolled novelties that extend menus contribute to the production of effort and regret emotions that, in turn, change the bargaining power between conflicted preferences. A rational individual, that anticipates the emotional consequences of uncontrollable menu extensions, will demand some flexibility as he chooses his menu, so as to allow himself the ability to adapt his consumption accordingly. However, by choosing greater flexibility additional costly emotions (regret and effort) are produced, so full flexibility ($m = 1$) could be suboptimal.

The second property states some intuitive comparative statics. The menu flexibility increases with p, the probability of facing new uncontrolled alternatives, δ, the individual discount factor and ρ, the impact of emotions.

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7 APPENDIX

Proof of Lemma 1

By the emotional balance transition equation and the fact that $x_t = \frac{1}{2} + \frac{\rho}{4} B_t$ we know that

$$B_{t+1} = \left(1 - \lambda - \frac{\rho}{2}\right) B_t + \mu.$$

Solving recursively, with the initial condition $B_1 \in \mathbb{R}$, we get

$$B_t = B_1 \left[1 - \lambda - \frac{\rho}{2}\right]^{t-1} + 2\mu \frac{1 - \left[1 - \lambda - \frac{\rho}{2}\right]^{t-1}}{2\lambda + \rho}.$$

Proof of Corollary 1

Immediate from Lemma 1. *Q.E.D*

Proof of Lemma 2

Consumption in period T is given by $x_T = \arg\max_{x \in M} W_T = \frac{1}{2} + \frac{\rho}{4} B_T$. The first order condition (FOC) of the maximization problem in period $t < T$ is given by

$$\frac{dW_t}{dx_t} = -4x_t + 2 + \rho B_t + \delta \frac{dW_{t+1}}{dx_t} = 0. \tag{2}$$

Plugging x_T in W_{T-1} and taking derivatives we get:

$$\frac{dW_T}{dx_{T-1}} = -\rho \left[\frac{\rho}{2} \left[(1-\lambda) B_{T-1} + \mu\right] - (1 - 2x_{T-1}) \left(1 - \frac{\rho}{2}\right)\right].$$

Solving the FOC (2) in period $T-1$ we obtain:

$$x_{T-1} = \frac{1}{2} + \frac{\rho \left(B_t \left(1 - (1-\lambda) \frac{\delta\rho}{2}\right) - \frac{\delta\rho}{2}\mu\right)}{4 + 2\delta\rho \left(1 - \frac{\rho}{2}\right)}.$$

If we keep moving backwards we get that for all $t < T$

$$x_t = \frac{1}{2} + \frac{\rho \left(B_t \left(1 - (1-\lambda) \frac{\delta\rho}{2} f_t\right) - \frac{\delta\rho}{2}\mu f_t - \delta (1 - f_t)(1 - 2x_{t+1})\right)}{4 + 2\delta\rho \left(1 - \frac{\rho}{2} f_t\right)}. \tag{3}$$

where $f_{T-1} = 1$ and $f_t = \frac{1 - (1-\lambda)\frac{\delta\rho}{2} f_{t+1}}{1 + \frac{\delta\rho}{2}\left(1 - \frac{\rho}{2} f_{t+1}\right)}$ for all $t < T - 1$.

Given the initial emotional balance, B_1, we can write emotional balance in period $t > 2$ as a function of previous consumption decisions:

$$B_t = (1-\lambda)^{t-1} B_1 - 2\left[x_{t-1} + (1-\lambda) x_{t-2} + ... + (1-\lambda)^{t-2} x_1\right] + (1+\mu)\left[\frac{1-(1-\lambda)^{t-1}}{\lambda}\right].$$

Therefore, we can rewrite (3) as:

$$\begin{aligned} x_1 &= \beta_1 x_2 + \gamma_1 \text{ and} \\ x_t &= \alpha_t \left[x_{t-1} + (1-\lambda) x_{t-2} + ... + (1-\lambda)^{t-2} x_1\right] + \beta_t x_{t+1} + \gamma_t \text{ for all } t \in \{2, ..., T-1\}. \end{aligned}$$

where $\alpha_t = -\frac{\rho\left(1-(1-\lambda)\frac{\delta\rho}{2} f_t\right)}{2+\delta\rho\left(1-\frac{\rho}{2} f_t\right)}$, $\beta_t = -\frac{\delta\rho(1-f_t)}{2+\delta\rho\left(1-\frac{\rho}{2} f_t\right)}$ and
$\gamma_t = \frac{1}{2} - \frac{\delta\rho\left(\frac{\rho}{2}\mu f_t - (1-f_t)\right)}{4+2\delta\rho\left(1-\frac{\rho}{2} f_t\right)} - \frac{\alpha_t}{2}\left((1-\lambda)^{t-1} B_1 + + (1+\mu)\left[\frac{1-(1-\lambda)^{t-1}}{\lambda}\right]\right)$.

The second order condition (SOC) of the maximization problem in period $t < T$ is given by

$$\frac{d^2 W_t}{d^2 x_t} = -4 - 2\delta\rho\left[1 - \frac{\rho}{2}\frac{1-(1-\lambda)\frac{\delta\rho}{2} f_t}{1+\frac{\delta\rho}{2}\left(1-\frac{\delta\rho}{2}\left(1-\frac{\rho}{2} f_t\right)\right)}\right] < 0$$

Therefore, a sufficient condition for SOC to be satisfied is $\rho \in [0, 2]$.

Q.E.D

Proof of Lemma 3

Immediate from Lemma 2. *Q.E.D*

Proof of Lemma 4

Using $x_1\left(\overline{M}\right)$ and $x_2\left(\overline{M}\right)$ we get $W_2\left(\overline{M}\right) = U\left(x_2\left(\overline{M}\right), e_2\left(x_1\left(\overline{M}\right)\right)\right) + V\left(x_2\left(\overline{M}\right), g_2\left(x_1\left(\overline{M}\right)\right)\right)$ and $W_1\left(\overline{M}\right) = U\left(x_1\left(\overline{M}\right), 0\right) + V\left(x_1\left(\overline{M}\right), 0\right) + \delta W_2\left(\overline{M}\right)$.

The menu that maximizes the individual's ex-ante surplus in $t = 0$ is given by the solution of the following problem:

$$m^* \in \arg\max_{m \in \left[\frac{1}{2}, 1\right]} \frac{p}{2} W_1(M \cup \{1\}) + \frac{p}{2} W_1(M \cup \{0\}) + (1-p) W_1(M)$$

Whose solution is:

$$m^* = 1 - \frac{|4 + \delta\rho(2-\rho)|}{\sqrt{p\rho\left(8 + \delta\rho\left(8 + 2\rho(1+\delta) + 3\delta\rho^2\right)\right)}}.$$

Therefore,

$$m^* > \frac{1}{2} \text{ iff } p > \overline{p}(\delta, \rho)$$

with

$$\overline{p}(\delta, \rho) = \frac{4\left(4 + \delta\rho\left(2 - \rho\right)\right)^2}{\rho\left(8 + \delta\rho\left(8 + 2\rho\left(1 + \delta\right) + 3\delta\rho^2\right)\right)}.$$

Q.E.D